M

To the Most Illustrious

JOSEPH SLOP *DE CADENBERG*,

Doctor of Laws, Professor of Astronomy in the University of Pisa, and Member of the Academy of Bononia.

Moſt Illuſtrious Sir,

THE other nations of Europe ought to regard Italy with an eye of particular gratitude. The Sciences and the fine Arts and Commerce, which Superſtition and the inundations of Barbarians had almoſt entirely deſtroyed, were revived

and greatly advanced by the noble efforts of your Countrymen.

'Tis to your GIOIA we owe the *Mariner's Compaſs*; to your COLOMBO, *America*; to your TORRICELLI, the *Barometer*, and to your great GALILEO, the *Law of the fall of Heavy Bodies.*

Italy can at this day boaſt of having given birth to a Mr de la G---GE, Director of a very celebrated Academy in Pruſſia, one of the moſt profound Mathematicians that ever appeared; and to a gentleman, greatly diſtinguiſhed by his ſcientific knowledge and his amiable

qualities, whom delicacy hinders me from naming.

I am happy in having this opportunity of acknowledging in a public manner the obligations which I contracted to you during my long ſtay at Piſa. Among the many very conſiderable marks of friendſhip with which you honoured me, that of directing my Mathematical Studies was none of the leaſt. For any little proficiency I may have made, I am indebted to You; and to You I think myſelf obliged in gratitude to dedicate the firſt fruits of my labours.

To merit the continuance of your friendſhip and eſteem ſhall be the invariable endeavour of one, who has the honour to be, with the greateſt reſpect,

Moſt Illuſtrious Sir,

Your moſt obliged,

And moſt devoted Servant,

Edinburgh,
Feb. 18. 1783.

WALTER MINTO.

P R E F A C E.

WE owe many of the moſt important Diſcoveries in Nature and Art to Chance, and to Men of Profeſſions totally unconnected with them.

Printing was invented by a Soldier, and Gunpowder by a Monk.

The Magnifying Powers of the Teleſcope have been greatly extended, and a NEW PLANET diſcovered, by Mr WILLIAM HERSCHEL, a Muſician by profeſſion.

We have no account of the Diſcovery of the other Planets. Being all viſible to the naked eye, their appearance and motions, different from thoſe of the fixed ſtars, would be remarked in the firſt ages of mankind.

The Diſcovery of the NEW PLANET will form an Epoch in Aſtronomy. Although it is inviſible to the naked eye, it may have been obſerved by different Aſtronomers ſince the invention of the Teleſcope; for it requires no great magnifying power to ſee it with ſufficient diſtinctneſs: But it differs little in appearance from ſome fixed ſtars of an inferior magnitude, and has been probably miſtaken for one of them.

While Mr HERSCHEL, engaged in a ſeries of Obſervations, contrived in a moſt ingenious manner, on the Parallax of the fixed ſtars, was examining the ſmall ſtars in the neighbourhood of H Geminorum, he perceived among them, in the evening of the 13th of March 1781, one which appeared viſibly larger than the reſt. Being ſtruck with its uncommon magnitude, he compared it to H Geminorum, and the ſmall ſtar in the quartile between Auriga and Gemini,

and finding it ſo much larger than either of them, ſuſpected it to be a *Comet.*

"I knew," ſays he, (PHIL. TRAN. 1781, *Part* II.) "from experience, that the "diameters of the fixed ſtars are not pro-"portionally magnified with higher pow-"ers as the planets are. I obſerved it "with the magnifying powers 227, 460, "and 932, and found the diameter of "the comet encreaſed in proportion."

All the aſtronomers of Europe were ſoon informed of the appearance of this New Star, and moſt of them have continued, as circumſtances allowed, to obſerve it ever ſince.

The ſlowneſs of its motion; its motion according to the order of the ſigns of the Zodiac; its vicinity to the ecliptic, and ſome other circumſtances, made them ſoon ſuſpect it to be a *Planet.*

A memoir was read to the Royal Academy of Sciences and Belles Lettres at Berlin relative to this New Star, ſeven or eight months after its diſcovery; and Mr De la Lande read a memoir on the ſame ſubject, in the courſe of laſt winter, to the Royal Academy of Arts and Sciences at Paris, at a meeting in which the Grand Duke and Ducheſs of Ruſſia were preſent.

But the only work, of any conſequence on this ſubject, that, as far as I know, has hitherto appeared, is the *Obſervationes et Theoria Novi Planetæ*, publiſhed at Piſa in the month of July laſt year by Mr Slop. On account of the very great accuracy of his aſtronomical obſervations, and the ſcrupulous exactneſs of his calculations, joined to a profound knowledge of geometry, this Gentleman is deſervedly ranked with the firſt of his profeſſion.

The following ſheets are divided into ſeven Sections.

The firſt Section contains part of the ſolution of the Problem, *To determine the orbit of a comet by three obſervations*, by Captain TEMPELHOFF of the King of Pruſſia's Artillery. This problem was propoſed a few years ago by the Royal Academy of Arts and Belles Lettres at Berlin, and premiums were adjudged to the Marquis de CONDORCET and to Mr TEMPELHOFF for their ſolutions of it.

From the firſt Section are deduced in the ſecond ſome methods for calculating the radii vectores of the Planets, the times of their periodical revolutions being ſuppoſed unknown. Theſe methods were tried by Mr SLOP and myſelf, for determining the orbit of the New Planet; but our ſucceſs did not at all anſwer our expectations: Our reſults in ſeveral different combinations differed widely from one another, and from the truth. They ſeem however not unworthy of a place, as ſome

of them may be of uſe to calculate the orbits of ſome Comets.

Mr SLOP, deſpairing of ſucceſs from any known methods, found out an equation, by means of which he determined, with great accuracy, the radius vector of the New Planet at the time of its oppoſition to the Sun on 21ſt December 1781. A deſcription of his equation, and the demonſtration of a ſimilar one are given in the third Section. In both theſe equations the radius vector is deduced from two obſervations, both very near, or one of them very near and the other at, the oppoſition. But as an error of even leſs than half a ſecond of a degree, in obſervations ſo very near each other, muſt make a conſiderable error in the value of the radius vector, theſe equations can ſeldom be uſed to advantage. Mr SLOP's ſucceſs depended on the great exactneſs of two of his own obſervations made in the neighbourhood of the oppoſition.

The two equations referred to led me to think, that by *any two* obſervations whatever the *circular* orbit of a Planet might be determined. This is demonſtrated in Section IV. The Reader will eaſily ſee that theſe two equations are corollaries which may be drawn from the ſolution of the general Problem.

The fifth Section contains a number of obſervations of the New Planet, by Mr HERSCHEL at Bath, Mr SLOP at Piſa, Mr Profeſſor ROBISON at Edinburgh, and others, from its firſt diſcovery on 13th March 1781 to 23d January 1783.

The obſervations of Mr HERSCHEL are calculated from the account which he has given of them in the Philoſophical Tranſactions. In that account the Planet is traced in its motion through a group of Teleſcopic Stars, of which he gives figures and a ſcale. The different figures are connected by three interpolations, and the calculation proceeds on the ſuppoſition, that on March 13th

the Right Aſcenſion of the Planet was 84° 0′ 0″, and its North Declination 23° 33′ 0″. I do not hear that any more exact determination has been given of theſe places.

The obſervations made at Mannheim by Meſſ. MAYER and KOENIG, Aſtronomers to the Elector Palatine, with an excellent mural quadrant of eight feet radius by *Bird*, ſeem, eſpecially that of 1ſt February 1782, to be affected with conſiderable errors.

The Right Aſcenſions and Declinations obſerved by Mr SLOP were deduced from a compariſon of the Planet with ſeveral teleſcopic ſtars in its neighbourhood. Thoſe from the 5th October 1781 to 14th March 1782 incluſive, that of the 9th October excepted, were made with a mural quadrant of ſix feet radius by *Siſſon*, and all the reſt with an excellent reflecting teleſcope on an equatorial machine by *Short*. The longitudes and latitudes were com-

puted ſeparately by Mr SLOP and myſelf, and, on a compariſon, were found to agree to $\frac{1}{10}$ of a ſecond, the longitude of 20th January excepted. The longitudes want the correction for the nutation.

The obſervations by Profeſſor ROBISON are the intervals, in mean time, between the tranſits of the planet, and of a teleſcopic ſtar, which he calls A, ſituated between η and μ Geminorum. It is found in the Britannic Catalogue, page 6th line 10th from the top. Its mean right aſcenſion 1ſt January 1783, is nearly 91° 24′ 58″, and its declination is 23° 39′ 17″ north. The ſecond column contains the difference of the declinations of the planet, and of H Geminorum. That very ingenious Gentleman does not conſider theſe obſervations as of great moment, being made with no other inſtrument than a reflecting teleſcope furniſhed with a micrometer. The poſition of the hora-

ry wire was determined by means of a ruler making a certain angle with it, and ſo fixed that its edge covered two known fixed ſtars.

This ſection contains alſo the Sun's places, and the logarithms of its diſtances from the Earth for the mean times of moſt of the obſervations made at Piſa.

The ſixth Section contains an illuſtration of the general problem of Section IV. by obſervations of the New Planet. From ſeveral different combinations the radius vector of its *circular* orbit, the time of its periodical revolution, and the inclination of its orbit to the ecliptic, are determined. It has been obſerved for too ſhort a time, to enable us to determine its *elliptical* orbit with any tolerable degree of accuracy.

The laſt ſection contains Mr SLOP's theory, and its agreement with his obſer-

vations. The longitudes are counted from the Planet's oppoſition to the Sun in the year 1781, December 21^{d} 18 $38' \ 50''$ mean time at Piſa.

I hear Mr HERSCHEL has, with the approbation of the Royal Society, given to the New Planet, the name *Georgium Sidus*. The encouragement his preſent Majeſty, by his beneficence and his own example, gives to aſtronomy, certainly entitles him more than any other living Sovereign to that honour. But it is not very probable that this name will be continued. The *Satellites* of *Jupiter* were named by their diſcoverer GALILEO, *Pianeti Medicei*, in honour of his patrons the MEDICI, a family, the memory of which will be revered as long as a taſte for the fine arts and the ſciences remains. This name, however, was diſcontinued.

Had Mr HERSCHEL or Mr SLOP given it the name of ſome of the ancient heathen

deities, it would have been, without heſitation, univerſally adopted. Among that number, *Minerva*, without all doubt, deſerved the pre-eminence. The planet Venus has very properly obtained that name from its beauty and brilliancy; and the planet Mars has been ſo called from its red colour. The New Planet, being a teleſcopic ſtar, may be ſaid to denote the modeſty of the Goddeſs of Wiſdom.

Aſtronomers have agreed to diſtinguiſh each planet by a particular mark. Thus the mark for Jupiter is ♃, and that for Venus ♀. I have taken the liberty of denoting the New Planet by ♅, a compound of the firſt and laſt letters of the ſurname of its Diſcoverer.

RESEARCHES

INTO SOME PARTS OF THE

THEORY

OF THE

PLANETS, &c.

SECTION I.

Description of part of the solution of the Problem, To determine the orbit of a Comet by three obſervations.

THREE obſervations of a planet or comet being given, and calling

Its curti-diſtances from the earth, at the firſt, ſecond and third obſervations x, y, and z.

Its geocentric latitudes λ, λ' and λ''.

Its elongations from the ſun e, e' and e''.

The earth's diſtances from the ſun a, b, and c,

And the ſun's motion from the firſt to the ſecond obſervation, and from the firſt to the third d and d'.

Mr Tempelhoff rigidly demonſtrates, that

$$\text{I. } (\text{Tang. } \lambda \text{ ſin. } \overline{e''-e'-d'+d} + \text{tang. } \lambda' \times \text{ ſin. } \overline{e-e''+d'} + \text{tang. } \lambda'' \text{ ſin. } \overline{e'-e-d})\, xyz + (\text{tang. } \lambda' \text{ ſin. } \overline{e''-d'} - \text{tang. } \lambda'' \text{ ſin. } \overline{e'-d})\, ayz + (\text{tang. } \lambda'' \text{ ſin. } \overline{e+d} - \text{tang. } \lambda \text{ ſin. } \overline{e''-d'+d})\, bxz + (\text{tang. } \lambda \text{ ſin. } \overline{e'+d'-d} - \text{tang. } \lambda' \times \text{ ſin. } \overline{e+d'})\, cxy - \text{tang. } \lambda'' \text{ ſin. } d.\, abz + \text{tang. } \lambda' \times \text{ ſin. } d'.\, acy - \text{tang. } \lambda \text{ ſin. } \overline{d'-d}.\, bcx = 0.$$

The ſectors deſcribed by the radius vector of a planet or comet are proportional to the times it takes to deſcribe them. Theſe ſectors, if the motion of the heavenly body is ſlow, or the times ſhort, will be

nearly proportional to triangles which have the ſame ſides with the ſectors, and the chords of their arcs for baſes. Any two of theſe contiguous triangles are proportional to the parts of the chord of the arc of the two correſpondent ſectors determined by the interſection of that chord, and the ſide common to both triangles: Or, in other words, if (fig. I.) C C′ C″ repreſent part of the orbit of a planet or comet, and SC, SC′, SC″ its radii vectores, drawing the chord CEC″, and calling the time between the firſt and ſecond obſervations and the time between the ſecond and third m and n, we ſhall have EC : EC″ :: m : n.

This granted, our author proves, that

$$\text{II. } \left(\frac{n}{m} a \text{ ſin. } \overline{e' - d} + c \text{ ſin. } \overline{e' + d' - d}\right) y - \frac{n}{m} \times$$

$$\text{ſin. } \overline{e' - c - d} . xy + \text{ſin. } \overline{e'' - e' - d' + d} . yz - \frac{n}{m} b \times$$

$$\text{ſin. } \overline{e + d} . x - b \text{ ſin. } \overline{e'' - d' + d} . z + b\left(\frac{n}{m} a \times\right.$$

$$\left.\text{ſin. } d - c \text{ ſin. } \overline{d' - d}\right) = 0$$

By a very ingenious analyſis, he ſhews likewiſe, that

III. $(\text{Sin. } e' \text{ tang. } \lambda'' - \text{ſin. } \overline{e'' - d' + d} \text{ tang. } \lambda') \cdot z - \frac{n}{m}\left(\text{ſin. } \overline{e+d} \text{ tang. } \lambda' - \text{ſin. } e' \text{ tang. } \lambda\right) x + \left(\frac{n}{m} a . \text{ſin. } d - c \text{ ſin. } \overline{d' - d}\right) \text{tang. } \lambda' = 0$

or expreſſing the known quantities by f, g, and h,

III. $fz - gx + h = 0$.

If we call the radii vectores R, R′ and R″ we have

IV. $R^2 = a^2 - 2ax \text{ coſin. } e + x^2 \text{ ſec.}^2 \lambda$.

V. $R'^2 = b^2 - 2by \text{ coſin. } e' + y^2 \text{ ſec.}^2 \lambda'$.

VI. $R''^2 = c^2 - 2cz \text{ coſin. } e'' + z^2 \text{ ſec.}^2 \lambda''$

SECTION II.

Some methods of determining the Radii vectores of the Planets, deduced from Sect. I.

IF the planet's orbit be ſuppoſed circular, the IV. V. and VI. equations, compared together, will give values for x and y, expreſſed by known quantities, and by z, which, ſubſtituted in equation I. or II. will free that equation from all unknown quantities, z excepted.

The value of z drawn from the III. equation, and ſubſtituted in equation VI. will, on a compariſon of equations IV. and VI. produce a quadratic equation, where x only is the unknown quantity.

Since any three obſervations, at ſhort intervals from each other, give

$$fz-gx+h=0,$$

it is evident that, if the firſt and laſt of theſe obſervations are ſuppoſed to remain the ſame, and the middle one to be changed,

$$f'z-g'x+h'=0;$$

four obſervations, therefore, will give the lineary equation,

$$x=\frac{fh'-f'h}{fg'-f'g}$$

If five obſervations are given: calling the correſpondent curti-diſtances from the earth r, s, t, u, v, and taking, inſtead of f, g and h, the letters p, q and k, we have from

The firſt, ſecond and third obſervations,

$$\text{I. } pt-qr+k=0;$$

The firſt, ſecond and fourth,

$$\text{II. } p'u-q'r+k'=0;$$

The firſt, ſecond and fifth,

$$\text{III. } p''v-q''r+k''=0.$$

The firſt of theſe equations, compared with the ſecond, gives

$$u = \frac{pq't + q'k - qk'}{p'q}$$

and the firſt with the third,

$$v = \frac{pq''t + q''k - qk''}{p''q}$$

If we now take the third, fourth, and fifth obſervations, and make $t=x$, $u=y$ and $v=z$, and ſubſtitute the values of the two laſt in equation I. of Sect. 1. we obtain a cubic equation, the ſolution of which will give the value of x. If the ſame values are ſubſtituted in equation II. of ſaid ſection, we have a quadratic equation.

When the ſecond of three obſervations is ſuppoſed made during the planet's oppoſition to the Sun, we have the angle $e' = 180°$, and, conſequently,

$$-\text{ſin.}\ \overline{e'' - d' + d}.\ z - \frac{n}{m}\text{ſin.}\ \overline{e + d}.\ x + \frac{n}{m} a\ \text{ſin.}\ d - c.\ \text{ſin.}\ \overline{d' - d} = 0,$$

an equation free from every function of the planet's latitude; a circumſtance well worth noting, eſpecially when that latitude is conſiderably ſmall; for a very little error in latitudes not ſurpaſſing a few minutes, muſt produce a very great one in an equation of this kind, where the tangents of theſe latitudes enter.

If therefore the ſecond of five obſervations is ſuppoſed that of the planet's oppoſition to the Sun, and the values of the curti-diſtances at the third, fourth and fifth ſubſtituted in equation II. (of the former ſection) which alſo is free of all functions of the planet's latitude, one may very well be tempted to hope for a tolerably exact reſult from the ſolution of a quadratic equation.

It is hardly worth while to obſerve, that the curti-diſtances from the Earth being found, it is eaſy to calculate the radii

vectores and all the other elements of the orbit of a planet.

SECTION III.

Description of Mr Slop's equation for finding the Radius vector of a Planet supposed to describe a circle, and the demonstration of another equation of the same nature.

MR SLOP takes two interpolated places, the one twelve hours before, and the other twelve hours after the planet's opposition to the Sun, calls

The radius vector x

The Sun's distance from the Earth at the time of the opposition a

The Sun's mean diurnal motion at that time m

The Earth's diurnal motion at that ſame time n

The planet's obſerved diurnal geocentric motion q

The planet's heliocentric latitude λ

And demonſtrates

$$x^3 - \frac{2a\,(n+q)\,x^2}{q.\,\text{coſin.}\,\lambda} + \frac{a^2(n+q)^2 x}{q^2\,\text{coſin.}^2\lambda} - \frac{m^2}{q^2} = 0$$

An equation of the ſame nature may be obtained in the following manner:

Suppoſe (fig. II.) the planet's place in the ecliptic at the time of its oppoſition to the ſun in the point E′, and its place a few hours before or after, in the point E: let the Sun be in S, and the Earth in the points T and T′, correſponding to the places of the planet.

From the point S, through T′ and E′, draw the indefinite ſtraight line SG. Join S and E by the ſtraight line SE, and S and T by ST. Draw TE and produce it till it meet SG in G. From the point T draw TF perpendicular to SG. Join the points E and E′; it is evident that EE′

will be ſenſibly a ſtraight line, and perpendicular to SG, conſequently parallel to TF. This done, let

The Sun's diſtance from the Earth, $ST = a$,

The radius vector $= x$,

The planet's heliocentric latitude $= \lambda$,

Its elongation, $STG = T$,

The Sun's motion, $TSG = V$,

And the time between the obſervations ſuppoſed expreſſed in decimals of a year $= t$,

We have

$SE = SE' = x$ coſin. λ,

and the time of the planet's periodical revolution being $x\sqrt{x}$,

The angle $ESE' = \frac{360°t}{x\sqrt{x}}$

and therefore

The line $EE' = \frac{360°t.\ \text{coſin.}\ \lambda}{\sqrt{x}}$

$TF = a$ ſin. V, and $SF = a$ coſin. V.

$$SG = \frac{a \text{ ſin. V.}}{\text{ſin. } \overline{T+V.}} = x \text{ coſin. } \lambda + GE'$$

$$GE' = \frac{a \text{ ſin. V.}}{\text{ſin. } \overline{T+V.}} - x \text{ coſin. } \lambda.$$

$$FG = SG - SF = \frac{a \text{ ſin. V.}}{\text{ſin. } \overline{T+V.}} - a \text{ coſin. V.}$$

The ſimilar triangles GE′E and GFT give

$$GE' : E'E :: GF : FT,$$

or ſubſtituting their values,

$$\frac{a \text{ ſin. V.}}{\text{ſin. } \overline{T+V.}} - x \text{ coſin. } \lambda : \frac{360^\circ . t. \text{ coſin. } \lambda}{\sqrt{x.}}$$

$$:: \frac{a \text{ ſin. V.}}{\text{ſin. } \overline{T+V.}} - a \text{ coſin. V} : a \text{ ſin. V.}$$

therefore,

$$\frac{a \text{ ſin. T} - \text{ſin. } \overline{T+V.} \text{ coſin. } \lambda . x}{\text{ſin. T} - \text{coſin. V. ſin. } \overline{T+V.}} = \frac{360^\circ . t. \text{ coſin. } \lambda}{\text{ſin. V} . \sqrt{x.}}$$

or making

$$\frac{a \text{ ſin. T.}}{\text{ſin. T} - \text{coſin. V. ſin. } \overline{T+V.}} = A.$$

$$\frac{\text{ſin. } \overline{T+V}.}{\text{ſin. } T - \text{coſin. } V.\ \text{ſin. } \overline{T+V}.} = B.$$

and

$$\frac{360^\circ t.}{\text{ſin. } V.} = C.$$

We have the cubic equation

$$x^3 - \frac{2\,A\,x^2}{B \text{ coſin. } \lambda} + \frac{A^2 x}{B^2 \text{ coſin}^2 \lambda} - \frac{C^2}{B^2} = 0,$$

ſimilar to that of Mr Slop, and anſwering the ſame purpoſe.

In this, as well as in Mr Slop's equation, the heliocentric latitude is ſuppoſed known. If that latitude is ſmall it may be taken at random. A firſt approximated value of x will give the neceſſary correction.

SECTION IV.

Solution of the Problem: To determine the circular orbit of a Planet from two obſervations.

LET the point S (fig. III.) repreſent the center of the Sun, the curve TT′ part of the Earth's orbit, the curve PP′ part of the orbit of a planet ſuppoſed to move in a circle, and the curve EE′ part of the ecliptic. While the earth is ſuppoſed in T, let the planet be ſuppoſed obſerved in P; and while the Earth is in T′ the Planet in P′. Draw the ſtraight lines ST, ST′, SP and SP′. From the points P and P′ let fall perpendicular to the plane of ecliptic the ſtraight lines PE and P′E′. Join the points S and E, S and E′, T and E, T′ and E′.

Let

The angles of elongation, $STE = T$, $ST'E' = T'$;

The angles formed by the interſections of the curti-diſtances from the Sun [SE, SE′], and the curti-diſtances from the Earth [TE, T′E′] in the points E and E′, $SET = E$, $SE'T' = E'$;

And the Planet's motion in the ecliptic in the interval of the obſervations $ESE' = \Sigma$.

Calling the Sun's longitude at the firſt obſervation $\odot$, and at the ſecond $\odot'$;

And the correſpondent geocentric longitudes of the Planet P and P′;

And making $P' - P = M$,

we have

$$M - E' + E - \Sigma = 0$$

For $T = \odot - P$, $T' = \odot' - P'$, the angle $EST = 180° - \odot + P - E$, $E'ST' = 180° - \odot' + P' - E'$, and the angle $T'ST = \odot' - \odot$, ſubſtituted in the ſelf-evident propoſition

$$EST' = EST - T'ST = E'ST' - E'SE$$

give

$$180° - \odot + P - E - \odot' + \odot = 180° - \odot' + P' - E' - \Sigma.$$

which reduced produces the above equation.

In order to find the expreſſions of ſome functions of the arcs or angles E, E' and Σ,

Let

The Earth's diſtance from the Sun at the firſt obſervation $= a$, and at the ſecond $= a'$.

The correſpondent heliocentric latitudes of the planet $= \lambda$ and λ'.

The radii vectores $SP = SP' = R$.

And the planet's motion in its orbit $PSP' = S$.

Join the points P and P', E and E' and parallel to EE' from the point P to where it meets $P'E'$, draw the ſtraight line PQ. PQ will be perpendicular to $P'E'$.

The right angled triangles PES and $P'E'S$ give

$$SE = R.\ \text{coſin.}\ \lambda\ ;\ \&\ SE' = R.\ \text{coſin.}\ \lambda'\ ;$$

And the plain triangles SET and SET′

$$\text{Sin.}E=\frac{a\ \text{fin. T.}}{\text{R.cofin.}\lambda,}\ \text{and fin. } E'=\frac{a'.\ \text{fin. T}'}{\text{R. cofin. }\lambda'}$$

$R\sqrt{R}$ being the time, expreffed in years and decimals of years, of the periodical revolution of the planet; if we call t, expreffed in the fame manner, the time between the two obfervations: we have

$$S=\frac{360°.\,t}{R\sqrt{R}.}$$

The triangle ESE′ gives

$$EE' : ES :: \text{fin. } \Sigma : \text{fin. } EE'S\,;$$

$$\text{And } EE' : E'S :: \text{fin. } \Sigma : \text{fin. } \overline{\Sigma + EE'S.}$$

therefore

$$\frac{\text{cofin. }\lambda}{\text{fin. EE}'\text{S}}=\frac{\text{cofin. }\lambda'}{\text{fin. }\overline{\Sigma+\text{EE}'\text{S}}}$$

hence a value for fin. EE′S, which fubftituted in the firft proportion, with the neceffary operations, produces

$$(EE')^2 = R^2\ (\text{cofin. } \lambda - 2\ \text{cofin. } \lambda.\ \text{cofin. } \lambda'. \times \text{cofin. } \Sigma + \text{cofin.}^2\ \lambda').$$

We have likewiſe

$$P'Q = P'E' - PE = R\,(\text{ſin.}\ \lambda' - \text{ſin.}\ \lambda) =$$

$$= \sqrt{(P'P)^2 - (E'E)^2}$$

and

$$PP' = \frac{R.\ \text{ſin.}\ S}{\text{ſin.}\ \overline{90^\circ - \tfrac{1}{2}S}} = 2\,.R.\ \text{ſin.}\tfrac{1}{2}S$$

therefore

$$4R^2.\,\text{ſin.}^2\,\tfrac{1}{2}S - R^2\,(\text{coſin.}^2\lambda - 2\ \text{coſin.}\,\lambda.\ \text{coſin.}\,\lambda'.\ \times \text{coſin.}\,\Sigma + \text{coſin.}^2\lambda') - R^2\ (\text{ſin.}\,\lambda' - \text{ſin.}\,\lambda)^2 = 0.$$

hence

$$\text{coſin.}\,\Sigma = \frac{1 - \text{ſin.}\,\lambda.\ \text{ſin.}\,\lambda' - 2\,\text{ſin.}^2\frac{180^\circ.\,t.}{R\sqrt{R.}}}{\text{coſin.}\,\lambda.\ \text{coſin.}\,\lambda'}$$

We have now got expreſſions for the ſines of E and E′, and for the coſine of Σ, in which the only unknown quantities are the radius vector and the ſines and coſines of the heliocentric latitudes. The whole difficulty is therefore to find the expreſſions of theſe functions of the heliocentric latitudes.

For this purpofe: Let

The geocentric latitudes of the Planet be l and l',

And its correfpondent curti-diftances from the Earth, TE and $T'E' = v$ and v'.

In the right angled triangle PES we have

$$PE = v.\ \text{tang.}\ l = R.\ \text{fin.}\ \lambda.$$

therefore

$$\text{fin.}\ \lambda = \frac{v}{R}.\ \text{tang.}\ l.$$

$$\text{And cofin.}\ \lambda = \sqrt{1 - \frac{v^2}{R^2}.\ \text{tang.}^2\ l}.$$

The triangle SET gives

$$SE : ST :: \text{fin.}\ T : \text{fin.}\ E;$$

$$\text{And}\ SE : TE :: \text{fin.}\ T : \text{fin.}\ \overline{T+E},$$

where fubftituting for cofin. λ, fin. E, and cofin. E their values, we obtain

$$R^2 = a^2 - 2av.\ \text{cofin.}\ T + v^2.\ \text{fec.}^2\ l,$$

which gives

$$v = \frac{a \,.\, \text{cosin. T.}}{\text{sec.}^2 l} \pm \sqrt{\frac{a^2 \text{cosin.}^2 \text{T}}{\text{sec.}^4 l} - \frac{a^2}{\text{sec.}^2 l} + \frac{\text{R}^2}{\text{sec.}^2 l}}$$

In the ſame manner it is demonſtrated, that

$$v' = \frac{a' \,.\, \text{cosin. T}'}{\text{sec.}^2 l'} \pm \sqrt{\frac{a'^2 \text{cosin.}^2 \text{T}'}{\text{sec.}^4 l'} - \frac{a'^2}{\text{sec.}^2 l'} + \frac{\text{R}^2}{\text{sec.}^2 l'}}$$

Theſe expreſſions ſubſtituted reſpectively in the values of the ſines and coſines of the heliocentric latitudes, and the new expreſſions reſulting ſubſtituted in the formulas

$$\frac{a \,.\, \text{sin. T}}{\text{R. cosin. } \lambda}; \qquad \frac{a' \,.\, \text{sin. T}'}{\text{R. cosin. } \lambda'}$$

and

$$\frac{1 - \text{sin. } \lambda \,.\, \text{sin. } \lambda' - 2 \text{ sin.}^2 \frac{180^\circ . t}{\text{R}\sqrt{\text{R}}}}{\text{cosin. } \lambda \,.\, \text{cosin. } \lambda'}$$

will give expreſſions for the ſines of E and E′, and for the coſine of Σ, entirely free of every unknown quantity, the radius vector excepted.

Now as

$$\text{ſin. }E=\frac{a \text{ ſin. } T}{\sqrt{R^2-v^2 \text{ tang}^2. l}}=E-\frac{E^3}{1.2.3}+\frac{E^5}{1.2.3.4.5}-\&c$$

$$\text{ſin. }E'=\frac{a' \text{ ſin. } T.}{\sqrt{R^2-v'^2 \text{ tang}^2. l'}}=E'-\frac{E'^3}{1.2.3}+\frac{E'^5}{1.2.3.4.5}-\&c.$$

$$\text{coſin. }\Sigma=\frac{R^2\left(1-\frac{vv'}{R^2}\text{tang.}l.\text{tang.}l'-2\text{ ſin.}^2\frac{180°t}{R\sqrt{R}}\right)}{\sqrt{(R^2-v^2\text{tang.}^2l)(R^2-v'^2\text{tang.}^2l')}}$$
$$=1-\frac{\Sigma^2}{1.2}+\frac{\Sigma^4}{1.2.3.4}-\&c.$$

it is evident no algebraic expreſſions of E, E', and Σ, can be had, except their fifth powers ſhould be quantities infinitely little; and conſequently the equation

$$M-E'+E-\Sigma=0$$

cannot be ſolved in a direct manner.

COROLLARIES.

If the angle $T=180°$, which is the caſe if the firſt obſervation of the planet

is ſuppoſed made during its oppoſition to the Sun, we have

$$M - E' - \Sigma = 0,$$

or if $T' = 180°$,

$$M + E - \Sigma = 0,$$

and if both $T = 180°$ and $T' = 180°$,

$$M - \Sigma = 0,$$

which is of itſelf ſelf-evident.

If the time between the obſervations is ſhort, and the planet's heliocentric latitude encreaſes or diminiſhes ſlowly, we have

$$\Sigma = S,$$

and therefore

$$M - E' + E - S = 0,$$

and if both obſervations are near the

planet's oppofition to the Sun, we have

$$\text{fin. } E = E = \frac{a \,.\, \text{fin. } T}{R \,.\, \text{cof. } \lambda}$$

and

$$\text{fin. } E' = E' = \frac{a' \,.\, \text{fin. } T'}{R \,.\, \text{cof. } \lambda}$$

and therefore

$$M - \frac{a' \text{ fin. } T' + a. \text{ fin. } T}{R. \text{ cof. } \lambda} - \frac{360^\circ . \, t}{R\sqrt{R}} = 0,$$

an equation of the third degree, the folution of which will give you a very exact value of R, provided (the obfervations being exact), you have firft an approximated value to calculate the cofine of the planet's heliocentric latitude by.

If the geocentric longitudes are fuppofed equal, we have M=o; and confequently

$$-E' + E - \Sigma = 0.$$

this is the caſe with ſome before and ſome after the planet's being ſtationary. And if the time between the obſervations is ſhort, the motion in latitude ſlow, and the planet at a diſtance great enough to make ſin. E=E, and ſin. E′=E, the equation becomes lineary,

$$R-\left(\frac{360°.\,t.\ \text{coſin.}\ \lambda}{a.\ \text{ſin.}\ T-a'\ \text{ſin.}\,T'}\right)^2=0.$$

To make uſe of the general equation

$$M-E'+E-\Sigma=0.$$

Subſtitute an arbitrary number for R in the equations

$$v=\frac{a.\ \text{ſin.}\ T}{\text{ſec.}^2\ l}\pm\sqrt{\frac{a^2\ \text{ſin.}^2\ T}{\text{ſec.}^4\ l}-\frac{a^2}{\text{ſec.}^2\ l}+\frac{R^2}{\text{ſec.}^2\ l}}$$

and

$$v'=\frac{a'\ \text{ſin.}\ T'}{\text{ſec.}^2 l'}\pm\sqrt{\frac{a'^2\ \text{ſin}^2\ T'}{\text{ſec.}^4\ l'}-\frac{a'^2}{\text{ſec.}^2 l'}+\frac{R^2}{\text{ſec.}^2 l'}}$$

and the values of v and v' thus found being ſubſtituted in the formulas

$$\frac{v}{R}\,.\ \text{tang.}\ l,\qquad \frac{v'}{R}\,.\ \text{tang.}\ l'$$

will give you a value for the ſines, and conſequently for the coſines of λ and λ': Then calculate the ſines of E and E′, and you have the correſpondent arcs. You may then calculate S, after which you will eaſily find the coſine, and conſequently the arc Σ.

Subſtitute the arcs E, E′ and Σ thus found in the general equation, which will become o if you have hit upon the true value of R, or will approach it in proportion as your ſuppoſed value approaches the real.

Suppoſe another value to R, and repeat the operation as before, and you will ſee which ſuppoſition leads you neareſt the truth.

Let the firſt arbitrary value of R be r, and the ſecond r', and the equation, inſtead of o, will, in the firſt caſe, become $=\omega$, for example, and in the ſecond $=\omega'$. I ſay,

If the difference between the firſt and ſecond values of R produces the difference $\omega - \omega'$ in the equation, what quantity to be ſubtracted from the firſt value of R will give the quantity ω.

Or, algebraically, calling the quantity ſought m,

$$r - r' : \omega - \omega' :: m : \omega$$

which gives

$$\mathrm{m} = \frac{\omega\,(r - r')}{\omega - \omega'}$$

$$\text{And } R = r - m = \frac{\omega r' - \omega' r}{\omega - \omega'}$$

This value of R will be much nearer the true than any of the two former.

One or two operations more of the ſame kind will bring you to a value for R as near the true as can be deſired.

If the latitude is only of a few minutes, ſome trouble may be ſaved by ſuppoſing

the heliocentric equal to the geocentric latitude, or even equal to o, in order to obtain a firſt approximated value for R.

The formula

$$\frac{360°.t}{R\sqrt{R}.}$$

will be expreſſed in ſeconds of a degree in reducing 360° to ſeconds. This gives

$$S=\frac{1296000''.t}{R\sqrt{R}}.$$

Let t'' be the number of ſeconds of time in t, we ſhall have t equal to t'' divided by the number of ſeconds in a year, or

$$t=\frac{t''}{31558151''}.$$

This ſubſtituted in the laſt expreſſion of S gives

$$S=\frac{1296000''.t''}{31558151''.R\sqrt{R}}=\frac{0,040167.t''}{R\sqrt{R}}$$

and therefore

$$\text{Log. } S=8,6134934+\text{Log. } t''-\text{Log. } R\sqrt{R}.$$

In order to find the place of the planet's Nodes, and the Inclination of its orbit to the ecliptic:

Let the portion of the planet's circular orbit, (fig. IV.) defcribed in the interval of the two obfervations, be PP′. Produce P′P indefinitely, and defcribe part of a great circle of the ecliptic K′KN meeting P′P in the point N. From the points of the planet's places, P and P′, let fall portions of great circles PK and P′K′, perpendicular to K′KN.

We have

$PK = \lambda$, $P'K' = \lambda'$, $KK' = \Sigma$,

P—E—KN=Longitude of the planet's node,

And PNK=Inclination of its orbit.

The fpherical triangles PNK and P′NK′ give

$1 : \text{fin. } KN :: \text{cotang. } \lambda : \text{cotang. } PNK$,

$1 : \text{fin. } \overline{\Sigma + KN} :: \text{cotang. } \lambda' : \text{cotang. } PNK$;

therefore

$$\text{cotang. } KN = \frac{\text{cotang. } \lambda - \text{cotang. } \lambda' \text{ cofin. } \Sigma}{\text{cotang. } \lambda' \text{ fin. } \Sigma.}$$

SECTION V.

Obfervations of the NEW PLANET : *Sun's Places and Logarithms of its diftances from the Earth at the times of fome of thefe obfervations.*

RIGHT Afcenfions and Declinations of the New Planet, obferved by

Mr HERSCHEL at Bath.

1781.

Temp. ver. Baln.				*Afc. Rect.* H			*Dec. Bor.* H		
d.	h.	′	″	°	′	″	°	′	″
Mar. 13.	10	30	0	84	0	0,0	23	33	0,0

Mr MASKELYNE at Greenwich.

Temp. ver. Gren.									
Mai. 28.				87	6	0,0	23	37	0,0

Mr SLOP at Pifa.

Temp. ver. Pif.									
Aug. 22.	16	12	4	92	11	12,8	23	40	13,3
23.	15	56	54	92	13	43,7	23	40	16,3
24.	16	5	57	92	16	8,7	23	40	21,7

1781.

	Temp. ver. Pis. d. h. ′ ″	Asc. Rect. H. ° ′ ″	Asc. Bor. H. ° ′ ″
Aug.	26. 15 48 46	92 20 40,6	23 40 12,2
	27. 14 16 1	92 22 46,0	23 40 8,0
	28. 14 32 52	92 25 6,2	23 40 10,0
	29. 14 2 26	92 27 13,0	23 40 9,8
	30. 13 57 35	92 29 14,3	23 40 8,4
Sep.	1. 14 35 13	92 33 29,1	23 40 3,2
	2. 14 8 9	92 35 30,0	23 40 2,8
	5. 13 57 52	92 41 3,9	23 40 2,0
	11. 13 52 20	92 50 53,9	23 39 58,0
	15. 13 39 41	92 56 28,3	23 39 57,5
	18. 12 38 27	93 0 6,0	23 40 3,2
	24. 12 53 49	93 5 39,5	23 40 0,8
	29. 12 38 34	93 8 49,2	23 40 3,0
Oct.	5. 17 23 43	93 10 41,7	23 40 13,1
	6. 17 20 5	93 10 46,7	23 40 8,7
	8. 17 12 45	93 10 48,4	23 40 8,8
	9. 12 13 0	93 10 46,3	23 40 14,4
	14. 16 50 33	93 8 54,7	23 40 21,3
	18. 16 35 23	93 7 17,2	23 40 25,9
	24. 16 12 24	93 2 42,1	23 40 38,9

Mess. MAYER and KOENIG at Mannheim.

	Temp. ver. Man.		
Nov.	4. 15 28 40	92 49 19,9	23 41 31,0

Mr MASKELYNE.

	Temp. ver. Gren.		
	8.	92 42 20,0	23 41 30,0

Mr Slop.

	Temp. ver. Pis.				*Asc. Rect.* ♅			*Dec. Bor.* ♅		
	d.	h	′	″	°	′	″	°	′	″
1781.										
Nov.	11.	10	36	3	92	37	20,0	23	41	22,2
Dec.	4.	13	19	4	91	44	39,1	23	42	36,2
	6.	13	9	58	91	39	21,8	23	42	41,9
	8.	13	0	51	91	33	59,4	23	42	51,9
	13.	12	37	58	91	20	14,6	23	43	3,1
	14.	12	33	22	91	17	25,6	23	43	6,4
	22.	11	56	23	90	54	57,3	23	43	15,1
	23.	11	51	50	90	52	8,8	23	43	15,3
	27.	11	33	22	90	40	57,6	23	43	20,9
	28.	11	28	47	90	38	15,6	23	43	23,5
1782.										
Jan.	1.	11	10	22	90	27	6,7	23	43	29,2
	2.	11	5	48	90	24	28,4	23	43	30,6
	4.	10	56	42	90	18	56,1	23	43	30,7
	6.	10	47	36	90	13	34,2	23	43	33,2
	8.	10	38	32	90	8	14,8	23	43	33,9
	9.	10	34	1	90	5	32,6	23	43	33,1
	11.	10	25	1	90	0	27,6	23	43	34,8
	12.	10	20	34	89	57	57,3	23	43	32,2
	13.	10	16	7	89	55	22,4	23	43	34,2
	14.	10	11	38	89	52	54,9	23	43	33,8
	15.	10	7	11	89	50	31,4	23	43	33,3
	17.	9	58	21	89	45	53,0	23	43	30,8
	19.	9	49	34	89	41	3,2	23	43	33,0

1782.	*Temp. ver.* Pis.				*Asc. Rect.* H			*Dec. Bor.* H		
	d.	h.	′	″	°	′	″	°	′	″
Jan.	20.	9	45	12	89	38	40,7	23	43	35,4
	22.	9	36	30	89	34	12,1	23	43	34,6
	28.	9	10	48	89	21	41,8	23	43	33,0
Feb.	1.	8	53	57	89	13	58,0	23	43	29,7
	4.	8	40	47	89	8	54,2	23	43	26,6
	11.	8	13	9	88	58	33,9	23	43	21,5
	15.	7	57	16	88	53	42,9	23	43	21,8
	16.	7	53	20	88	52	37,0	23	43	23,2
	18.	7	45	31	88	50	44,0	23	43	21,8
	19.	7	41	37	88	49	47,1	23	43	21,3
	20.	7	37	45	88	48	53,5	23	43	17,7
	26.	7	14	48	88	44	58,6	23	43	15,8
	27.	7	11	2	88	44	41,5	23	43	11,2
Mar.	3.	6	56	5	88	43	21,9	23	43	12,3
	4.	6	52	22	88	43	12,4	23	43	10,7
	5.	6	48	41	88	43	6,9	23	43	11,9
	6.	6	44	58	88	43	7,8	23	43	10,1
	8.	6	37	36	88	43	3,6	23	43	8,2
	10.	6	30	16	88	43	28,6	23	43	7,9
	11.	6	26	37	88	43	44,0	23	43	10,6
	13.	6	19	23	88	44	28,3	23	43	10,4
	14.	6	15	45	88	44	55,2	23	43	11,9

Mr MASKELYNE.

Temp. ver. Gren.				
Oct. 24.	97 59	4,0	23 33	37,0

The geocentric Longitudes and Latitudes of the NEW PLANET, computed from the Right Afcenfions and Declinations obferved by

Mr HERSCHEL.

1781.

	Temp. ver. Baln.				*Long. geoc.* ♅			*Lat. Bor.* ♅		
	d.	h.	′	″	°	′	″	°	′	″
Mar.	13.	10	30	0	84	30	5,0	0	11	42,0
	17.	11	0	0	84	32	53,0			
	21.	10	0	0	84	35	58,0			
	24.	8	12	0	84	39	15,0			
	26.	10	43	0	84	41	28,0			
	28.	7	46	0	84	44	5,0			
	29.	8	50	0	84	45	33,0			
Apr.	1.	7	45	0	84	49	55,0			
	15.	10	18	0	85	13	45,0			
	16.	10	47	0	85	15	27,0			
	19.	8	18	0	85	22	11,0	0	13	33,0

Mr DE CESARIS at Milan.

	Temp. med. Pif.									
Mai.	15.	8	5	52	86	37	46,8	0	11	36,7

Mr MASKELYNE.

	Temp. ver. Gren.									
	28.				87	20	35,0	0	10	25,0

Mr Messier at Clugny.

1781.	*Temp. med.* Pis. d.	h.	′	″	*Long. geoc.* ♅ °	′	″	*Lat. Bor.* ♅ °	′	″
July	17.	15	49	15	89	42	22,0	0	12	1,3
				Mr Slop.						
Aug.	22.	16	14	21	92	0	10,2	0	12	55,5
	23.	15	58	55	92	2	28,4	0	13	0,6
	24.	16	7	42	92	4	41,1	0	13	8,1
	26.	15	49	58	92	8	50,2	0	13	2,6
	27.	14	16	57	92	10	45,1	0	13	0,2
	28.	14	33	30	92	12	53,5	0	13	4,4
	29.	14	1	47	92	14	49,6	0	13	6,2
	30.	13	56	38	92	16	40,7	0	13	6,7
Sept.	1.	14	34	38	92	20	34,1	0	13	5,5
	2.	14	7	16	92	22	25,1	0	13	7,1
	5.	13	55	59	92	27	30,6	0	13	12,0
	11.	13	48	24	92	36	31,0	0	13	18,4
	15.	13	34	22	92	41	37,2	0	13	24,0
	18.	12	32	7	92	44	56,4	0	13	33,8
	24.	12	45	25	92	50	1,9	0	13	37,9
	29.	12	28	31	92	52	55,5	0	13	43,8
Oct.	5.	17	11	46	92	54	38,3	0	13	56,1
	6.	17	7	51	92	54	43,1	0	13	51,8
	8.	16	59	58	92	54	44,6	0	13	51,9
	9.	12	0	1	92	54	42,5	0	13	57,5
	18.	16	20	25	92	51	30,8	0	14	4,8
	24.	15	56	36	92	47	18,6	0	14	12,4

Meff. MAYER and KOENIG.

1781.

	Temp. ver. Man.	*Long. geoc.* ♅	*Lat. Bor.* ♅
	d. h. ′ ″	° ′ ″	° ′ ″
Nov.	4. 15 28 40	92 35 4,0	0 14 48,0

Mr MASKELYNE.

	Temp. ver. Gren.		
	8.	92 28 39,0	0 14 43,0

Mr SLOP.

	Temp. med. Pif.		
	11. 10 20 26	92 24 4,1	0 14 28,1
Dec.	4. 13 10 6	91 35 49,0	0 14 58,3
	6. 13 1 51	91 30 58,4	0 15 0,5
	8. 12 53 37	91 26 3,2	0 15 7,1
	13. 12 32 5	91 13 27,9	0 15 10,8
	14. 12 28 58	91 10 53,1	0 15 12,7

Meff. MAYER and KOENIG.

	Temp. ver. Man.		
	21. 12 1 12	90 53 8,0	0 15 30,0

Mr Slop.

1781.	*Temp. med.* Pif.				*Long. geoc.* ♅			*Lat. Bor.* ♅		
	d.	h.	′	″	°	′	″	°	′	″
Dec.	22.	11	55	58	90	50	18,7	0	15	11,9
	23.	11	51	55	90	47	44,5	0	15	11,1
	27.	11	35	25	90	37	30,0	0	15	13,4
	28.	11	31	19	90	35	1,6	0	15	15,3
1782.										
Jan.	1.	11	14	48	90	24	49,2	0	15	18,7
	2.	11	10	3	90	22	24,3	0	15	19,6
	4.	11	2	31	90	17	20,1	0	15	18,9
	6.	10	54	18	90	12	25,4	0	15	20,9
	8.	10	46	5	90	7	32,9	0	15	21,2
	9.	10	41	58	90	5	4,5	0	15	20,3
	11.	10	33	46	90	0	25,2	0	15	21,9
	12.	10	29	41	89	58	7,6	0	15	19,1
	13.	10	25	37	89	55	45,8	0	15	21,4
	14.	10	21	29	89	53	30,8	0	15	21,0
	15.	10	17	23	89	51	19,5	0	15	20,7
	17.	10	9	13	89	47	4,6	0	15	18,6
	19.	10	1	3	89	42	39,3	0	15	21,3
	20.	9	56	58	89	40	28,8	0	15	24,0
	22.	9	48	48	89	36	22,9	0	15	23,9
	28.	9	24	23	89	24	56,0	0	15	24,7

Meff. Mayer and Koenig.

	Temp. ver. Man.									
Feb.	1.	8	54	7	88	55	2,0	0	15	38

Mr Slop.

1782.

	Temp. med. Pis.				*Long. geoc.* ♅			*Lat. Bor.* ♅		
	d.	h.	′	″	°	′	″	°	′	″
Feb.	1.	9	8	6	89	17	51,5	0	15	23,5
	4.	8	55	14	89	13	13,3	0	15	22,0
	11.	8	27	49	89	3	45,4	0	15	20,6
	15.	8	11	48	88	59	18,9	0	15	22,8
	16.	8	7	47	88	58	18,7	0	15	24,6
	18.	7	59	48	88	56	35,3	0	15	24,0
	19.	7	55	48	88	55	43,1	0	15	24,0
	20.	7	51	49	88	54	53,9	0	15	20,7
	26.	7	27	57	88	51	18,9	0	15	20,7
	27.	7	24	0	88	51	3,2	0	15	16,2
Mar.	3.	7	8	13	88	49	50,4	0	15	18,0
	4.	7	4	16	88	49	41,6	0	15	16,5
	5.	7	0	21	88	49	36,7	0	15	17,7
	6.	6	56	24	88	49	37,4	0	15	15,8
	8.	6	48	32	88	49	33,6	0	15	14,0
	10.	6	40	40	88	49	56,5	0	15	13,5
	11.	6	36	45	88	50	10,6	0	15	16,1
	13.	6	28	58	88	50	51,1	0	15	15,5
	14.	6	25	3	88	51	15,8	0	15	16,8
	Temp. ver. Pis.									
Aug.	20.	14	13	25	96	11	49,9	0	16	6,8
	21.	14	27	40	96	14	22,2	0	15	57,7
Sept.	3.	14	41	46	96	43	37,1	0	16	13,9
	4.	14	20	23	96	45	32,8	0	16	16,4

1782. *Temp. ver.* Pif. d. h. ′ ″					*Long. geoc.* ♅ ° ′ ″			*Lat. Bor.* ♅ ° ′ ″		
Sept.	12.	13	34	3	96	59	26,3	0	16	20,8
	13.	13	21	27	97	1	0,5	0	16	36,1
	25.	13	5	14	97	15	20,1	0	16	54,9
	26.	13	32	47	97	16	32,4	0	17	2,0

Mr MASKELYNE.

Temp. ver. Gren.						
Oct. 24.	97	18	55,0	0	17	36,0

Differences in mean time of the Tranfit of A Geminorum and the NEW PLANET, and differences of its Declination and that of H Geminorum, obferved at Edinburgh by

Mr Profeffor ROBISON.

1782. *Tem. med.* Edin. d. h. ′ ″					*Tr.* ♅ *poft* A. h. ′ ″			*D.* ♅—*D.* H ° ′ ″		
Dec.	4.	8	51	20	0	21	25	0	22	3
	9.	9	12	18	0	20	39			
	14.	8	11	43	0	19	56			
	15.	8	20	4	0	19	35			
	18.	10	2	14	0	19	4			

1782.

		Tem. med. Edin.			Tr.H. post A.			D.H.—D.H		
	d.	h.	′	″	h	′	″	°	′	″
Dec.	21.	7	58	51	0	18	30.	0	24	18
	24.	9	24	0	0	17	57			
	26.	8	41	10	0	17	35			
	28.	8	34	35	0	17	13			
	30.	7	48	17	0	16	51			
	31.	9	52	8	0	16	40			
1783.										
Jan.	1.	6	58	42	0	16	31			
	2.	8	18	29	0	16	17			
	5.	10	15	15	0	15	44	0	25	40
	8.	8	53	42	0	15	09			
	10.	8	51	33	0	14	49			
	11.	9	27	46	0	14	$37\frac{1}{2}$			
	18.	8	36	0	0	13	26			
	19.	10	26	30	0	13	16			
	23.	8	12	39	0	12	39			

Longitudes of the Sun and Logarithms of its diſtances from the Earth for the mean times of the obſervations made at Piſa from the 22d Auguſt 1781 to 14th March 1782, incluſive.

1781. d.	Long. ☉ s ° ′ ″	Log. diſt. ☉ a ♁
Aug. 22.	5 0 16 5,3	0,0045029
23.	5 1 13 25,0	0,0044036
24.	5 2 12 44,7	0,0043016
26.	5 4 7 2,2	0,0040953
27.	5 5 1 17,4	0,0039945
28.	5 6 0 2,6	0,0038874
29.	5 6 56 52,9	0,0037806
30.	5 7 54 46,8	0,0036735
Sept. 1.	5 9 52 33,0	0,0034539
2.	5 10 49 37,4	0,0033447
5.	5 13 43 53,2	0,0030160
11.	5 19 33 57,3	0,0023435
15.	5 23 27 42,1	0,0018793
18.	5 26 21 16,2	0,0015207
24.	6 2 14 51,9	0,0007653
29.	6 7 8 9,4	0,0001290
Oct. 5.	6 13 15 50,4	9,9993484
6.	6 14 14 58,9	9,9992284
8.	6 16 13 23,1	9,9989857
9.	6 17 0 25,7	9,9988880
18.	6 26 7 43,8	9,9977754
24.	7 2 5 53,3	9,9970391

1781.	d.	*Long.* ☉ s	°	′	″	*Log.dist.* ☉ *a* ♅
Nov.	11.	7	19	55	28,1	9,9951538
Dec.	4.	8	13	19	49,2	9,9933501
	6.	8	15	21	19,1	9,9932491
	8.	8	17	22	59,2	9,9931604
	13.	8	22	27	27,6	9,9929605
	14.	8	23	28	16,4	9,9929267
	22.	9	1	36	13,5	9,9927045
	23.	9	2	37	12,5	9,9926869
	27.	9	6	41	5,9	9,9926334
	28.	9	7	42	4,2	9,9926287
1782.						
Jan.	1.	9	11	45	57,0	9,9926351
	2.	9	12	46	56,2	9,9926430
	4.	9	14	48	52,7	9,9926708
	6.	9	16	50	49,7	9,9927074
	8.	9	18	52	54,1	9,9927530
	9.	9	19	53	46,4	9,9927789
	11.	9	21	55	33,5	9,9928336
	12.	9	22	56	41,7	9,9928640
	13.	9	23	57	39,5	9,9928974
	14.	9	24	58	36,7	9,9929310
	15.	9	25	59	33,2	9,9929656
	17.	9	28	1	24,3	9,9930392
	19.	10	0	3	11,4	9,9931285
	20.	10	1	4	3,2	9,9931632
	22.	10	3	5	43,6	9,9932556
	28.	10	9	10	16,2	9,9935873
Feb.	1.	10	13	12	56,0	9,9938633

F

1782.		Long. ☉	Long. dist. ☉ at
	d.	s ° ′ ″	
Feb.	4.	10 16 14 44,0	9,9940954
	11.	10 23 18 28,4	9,9946902
	15.	10 27 20 6,6	9,9950537
	16.	10 28 20 26,8	9,9951466
	18.	11 0 21 1,3	9,9953337
	19.	11 1 21 15,6	9,9954298
	20.	11 2 21 28,4	9,9955273
	26.	11 8 22 0,5	9,9961429
	27.	11 9 21 59,2	9,9962515
Mar.	3.	11 13 21 36,6	9,9967037
	4.	11 14 21 26,6	9,9968212
	5.	11 15 21 15,6	9,9969400
	6.	11 16 21 2,4	9,9970599
	8.	11 18 20 32,2	9,9973011
	10.	11 20 19 55,6	9,9975436
	11.	11 21 19 34,7	9,9976643
	13.	11 23 18 47,6	9,9979059
	14.	11 24 18 21,3	9,9980268

SECTION VI.

The general Problem of Section IV. *Exemplified in the* New Planet.

To illuſtrate our Problem, let us take the firſt and the laſt of Mr Slop's obſervations made at the Mural Quadrant.

The mean times at Piſa were

		d.	h.	′	″
1781.	October	5.	17	11	46.
1782.	March	14.	6	25	3.

The interval between the obſervations is therefore

d.	h.	′	″
159	13	13	17,

which reduced to ſeconds gives

$$t''=13785197''$$

$$\text{Log.}\ t''=7{,}1394131.$$

The Sun's places at the above periods were

$$\odot = \overset{s}{6}\ \overset{\circ}{13}\ \overset{\prime}{15}\ \overset{\prime\prime}{50{,}4};$$

$$\odot' = 11\ 24\ 18\ 21{,}3.$$

And ſuppoſing the Earth's mean diſtance from the Sun=1, the logarithms of the Earth's diſtances from the Sun at the ſame periods are,

$$\text{Log. } a = 9{,}9993484;$$

$$\text{Log. } a' = 9{,}9980268.$$

The geocentric Longitudes of the planet corrected by the nutation were

$$P = \overset{s}{3}\ \ \overset{\circ}{2}\ \overset{\prime}{54}\ \overset{\prime\prime}{30{,}4};$$

$$P' = 2\ 28\ 51\ 10{,}4;$$

hence the Planet's Elongations

$$T = \odot - P = \overset{s}{3}\ \overset{\circ}{10}\ \overset{\prime}{21}\ \overset{\prime\prime}{20{,}0};$$

$$T' = \odot' - P' = 8\ 25\ 27\ 10{,}9.$$

The logarithms of the ſines of theſe elongations are

$$\text{Log. ſin. } T = 9{,}9928676;$$

$$\text{Log. ſin. } T' = 9{,}9986310.$$

The planet's motion in geocentric longitude was

$$M = P' - P = 4^{\circ}\ 3'\ 20{,}0''.$$

The planet's geocentric Latitudes were

$$l = 0^{\circ}\ 13'\ 56{,}1'';$$

$$l' = 0\ 15\ 16{,}8.$$

The logarithms of the tangents and of the ſquares of the ſecants of theſe latitudes are

$$\text{Log. tang. } l = 7{,}6077645;$$

$$\text{Log. tang. } l' = 7{,}6476682;$$

$$\text{Log. ſec.}^2\ l = 10{,}0000071;$$

$$\text{Log. ſec.}^2\ l' = 10{,}0000083.$$

The geocentric latitudes being ſmall, the coſines of the correſpondent heliocentric latitudes will differ little from one another and from unity. To ſave trouble, therefore, we ſhall, in order to have a firſt approximated value for R, ſuppoſe coſin. λ=coſin.λ'=1, and Σ=S.

This gives us

$$\text{Log. ſin. E}=\text{log.}\ \frac{a\ \text{ſin. T}}{R}=9{,}9922160-\text{log. R.}$$

$$\text{Log. ſin. E}'=\text{log.}\ \frac{a'\ \text{ſin. T}'}{R}=9{,}9966578-\text{log. R.}$$

$$\text{Log. S}=\text{log.}\ \frac{0{,}040167.t''}{R\sqrt{R}}=5{,}7529065-\text{log. }R\sqrt{R}.$$

The quantities M and t'' compared together, ſhew that our planet muſt be far beyond the moſt diſtant of any with which we are acquainted in our ſyſtem. Let us begin then with ſuppoſing the radius vector R=15 times the Sun's mean diſtance from the Earth, which is about one third

greater than the diſtance of Saturn. We have

$$\text{Log. } R \quad = 1{,}1760913;$$

$$\text{Log. } R\sqrt{R} = 1{,}7641369;$$

conſequently

$$\text{Log. ſin. } E = 8{,}8161247.$$

$$\text{Log. ſin. } E' = 8{,}8205665.$$

$$\text{Log. } S \quad = 3{,}9887696.$$

therefore

$$\begin{array}{rrr} & {}^{\circ} & {}' \\ +M = & -4 & 3 \\ -E' = & +3 & 47 \\ +E = & +3 & 45 \\ -S = & -2 & 42 \\ \hline \omega = & +0 & 47 \end{array}$$

here we are far from the truth, for the ſum ω of the terms of our equation, inſtead of being $=0$ is $=47'$. It is evident, therefore, that R has been taken by far too little. Let us now ſuppoſe

$$R = 20,$$

we have

Log. R $=1,3010300.$

Log. $R\sqrt{R}=1,9515450.$

Log. ſin. $E=8,6911860.$

Log. ſin. $E'=8,6956278.$

Log. S $=3,8013615.$

therefore

	°	′	″
$+M=$	-4	3	20
$-E'=$	$+2$	50	38
$+E=$	$+2$	48	54
$-S=$	-1	45	29
$\omega=$	-0	9	17

here it is plain R has been taken too great, but that it is nearer the truth than the former value, becauſe $9' \; 17''$ are nearer to o than $47'$. Let

$R=19,05$

we have

Log. R $=1,2798950.$

Log. $R\sqrt{R}$=1,9198425.

Log. ſin. E=8,7123210.

Log. ſin. E'=8,7167628.

Log. S =3,8330640.

therefore

	°	′	″
+M=	−4	3	20
−E'=	+2	59	9
+E =	+2	57	20
−S =	−1	53	29
ω' =	−0	0	20

We are now very near an approximated value for R. In order to come at it the ſooner, I ſay: If the difference between the two laſt values of R, (0,95), give the difference between the two laſt values of ω, (8′ 57″); what quantity, *m*, to be ſubtracted from the penult value of R, will give the penult value of ω, (9′ 17″)? I find *m*=0,985, &c. and therefore

R=19,0155178.

Log. R=1.2791095.

Log. $R\sqrt{R}$=1,9186643.

Log. ſin. E =8,7131065.

Log. ſin. E′=8,7175483.

Log. S =3,8342422.

therefore

	°	′	″
+M=—4		3	20
—E′=+2		59	28
+E =+2		57	39
—S=—1		53	47
ω =	0	0	0

With this value of R I find the Planet's aberration at the firſt obſervation = —0″,5, and at the ſecond = —2″,6, and therefore

	s	°	′	″
P =	+3	2	54	29,9
P′=	+2	28	51	7,8
M=	—0	4	3	22,1
T =	+3	10	21	20,5
T′=	+8	24	27	13,5

$$\text{Log. fin. T} = 9{,}9928674.$$

$$\text{Log. fin. T}' = 9{,}9986314.$$

$$\text{Log. cofin. T} = 9{,}2546897.$$

$$\text{Log. cofin. T}' = 8{,}8990730.$$

$$\text{Log.} \frac{a \text{ cofin. T}}{\text{fec.}^2 l} = 9{,}2540410;$$

$$\text{Log.} \frac{a^2 \text{ cofin.}^2 \text{ T}}{\text{fec.}^4 l} = 8{,}5080820;$$

$$\text{Log.} \frac{a^2}{\text{fec.}^2 l} = 9{,}9986997;$$

$$\text{Log.} \frac{\text{R}^2}{\text{fec.}^2 l} = 2{,}5582219;$$

$$\frac{a \text{ cofin. T}}{\text{fec.}^2 l} = -0{,}1794903;$$

$$\frac{a^2 \text{ cofin.}^2 \text{ T}}{\text{fec.}^2 l} = +0{,}0322168.$$

$$\frac{a^2}{\text{fec.}^2 l} = +0{,}9970105;$$

$$\frac{R^2}{\text{sec.}^2 l}=+361,5945100.$$

therefore

$$\text{Log. } v=1,2744064.$$

$$\text{Log.} \frac{a' \text{ cosin. } T'}{\text{sec.}^2 l} =8,8971015;$$

$$\text{Log.} \frac{a'^2 \text{ cosin}^2 T'}{\text{sec.}^4 l'} =7,7942030;$$

$$\text{Log.} \frac{a'^2}{\text{sec.}^2 l'} =9,9960553;$$

$$\text{Log.} \frac{R^2}{\text{sec.}^2 l'} =2,5582207.$$

$$\frac{a' \text{ cosin. } T'}{\text{sec. } l'} =-0,0789045;$$

$$\frac{a'^2 \text{ cosin.}^2 T'}{\text{sec.}^4 l'} =+0,0062259;$$

$$\frac{a'^2}{\text{sec.}^2 l'} =+0,9909580;$$

$$\frac{R^2}{\text{sec.}^2 l'} =+361,5935100.$$

therefore

$$\text{Log. } v'.=1,2767107.$$

hence

$$\text{Log.}\frac{v}{R}\text{tang. } l = \text{Log. fin. } \lambda = 7,6030614,$$

$$\text{Log.}\frac{v'}{R}\text{tang. } l' = \text{Log. fin. } \lambda' = 7,6452694;$$

$$\text{Log. cofin. } \lambda = 9,9999965,$$

$$\text{Log. cofin. } \lambda' = 9,9999958.$$

Now, as we know that R is very near the truth, and as v and v' encreafe nearly in proportion to it, it is evident that a fmall alteration in the logarithms of the fines of λ and λ', will not fenfibly alter the fums of thefe logarithms, nor the logarithms of the cofines. We have therefore, as conftant quantities,

$$\text{Log.}\frac{1}{\text{cofin. } \lambda \text{. cofin. } \lambda'} = 0,0000077,$$

$$\text{Log.}\frac{\text{fin. } \lambda \text{. fin. } \lambda'}{\text{cofin. } \lambda \text{. cofin. } \lambda'} = 5,2483308,$$

$$\frac{1}{\text{cosin. } \lambda. \text{ cosin. } \lambda'} = 1,0000178,$$

$$\frac{\text{sin. } \lambda. \text{ sin. } \lambda'}{\text{cosin. } \lambda. \text{ cosin. } \lambda'} = 0,0000177.$$

Taking now the corrected values of T and T, we have

$$\text{Log. sin. E} = \text{Log.} \frac{a. \text{ sin. T}}{\text{R. cos. } \lambda} = 9,9922193 - \text{Log. R}$$

$$\text{Log. sin. E}' = \text{Log.} \frac{a'. \text{ sin. T}'}{\text{R. cos. } \lambda'} = 9,9966624 - \text{Log. R}$$

and, as before,

$$\text{Log. S} = 5,7[illegible]29065 - \text{Log. R}\sqrt{\text{R}};$$

M being a little greater than it was before, R will be a little less than we found it.

Let

$$\text{R} = 19,015,$$

we have

$$\text{Log. R} = 1,2790963,$$

$$\text{Log. R}\sqrt{\text{R}} = 1,9186444,$$

Log. sin. $E = 8,7131230$,

Log. sin. $E' = 8,7175661$,

Log. $S = 3,8342621$,

$S = 6827'',51$,

$\frac{1}{2}S = 3413'',75 = 0° 56' 53'',75$,

Log. sin. $\frac{1}{2}S = 8,2187804$,

$$\text{Log.} \frac{2. \text{ sin.}^2 \frac{1}{2}S}{\text{cosin. } \lambda. \text{ cosin. } \lambda'} = 6,7385985,$$

$$\frac{2 \text{ sin.}^2 \frac{1}{2}S}{\text{cosin. } \lambda. \text{ cosin. } \lambda'} = 0,0005477,7,$$

$$\text{Cosin.} \Sigma = \frac{1 - \text{sin. } \lambda. \text{ sin. } \lambda' - 2 \text{ sin.}^2 \frac{1}{2}S}{\text{cosin. } \lambda. \text{ cosin. } \lambda'} = 0,9994523,2.$$

therefore

	°	′	″
$+M =$	-4	3	22,10
$-E' =$	$+2$	59	29,20
$+E =$	$+2$	57	39,43
$-\Sigma =$	-1	53	46,75
$\omega =$	-0	0	0,22

R is still too great by a very small quantity. Let

$R=19{,}0144,$

$\text{Log. } R=1{,}2790826,$

$\text{Log. } R\sqrt{R}=1{,}9186239,$

we have

$\text{Log. fin. } E=8{,}7131367,$

$\text{Log. fin. } E'=8{,}7175798;$

$\text{Log. } S=3{,}8342826,$

$S=6827'',83,$

$\tfrac{1}{2}S=3413{,}92=0^\circ\,56'\,53'',92,$

$\text{Log. fin. } \tfrac{1}{2}S=8{,}2188017.$

$$\text{Log.}\frac{2.\ \text{fin.}^2\ \frac{1}{2}S}{\text{cofin. } \lambda.\ \text{cofin. } \lambda'}=6{,}7386411,$$

$$\frac{2.\ \text{fin.}^2\ \frac{1}{2}S}{\text{cofin. } \lambda.\ \text{cofin. } \lambda'}=0{,}0005478{,}24.$$

$\text{Cofin. } \Sigma=0{,}9994522{,}6.$

therefore

$$\begin{array}{rrrr} & {}^{\circ} & {}' & {}'' \\ +M= & -4 & 3 & 22,10 \\ -E'= & +2 & 59 & 29,54 \\ +E= & +2 & 57 & 39,76 \\ -\Sigma= & +1 & 53 & 47,11 \\ \hline \omega= & +0 & 0 & 0,09 \end{array}$$

R therefore lies between the narrow limits of 19,015 and 19,0144. By means of a proportion, I find

$$R=19,0144342.$$

I have therefore as follows:

Log R = .2790834;

Log. $R\sqrt{R}$ = .9186251;

Log. sin. E = 8,7131359;

Log. sin. E′ = 8,7 [illegible] 790;

Log. S = 3,8342814;

S = 6827″,81;

$$\tfrac{1}{2}S = 3413'',9 = 0^{\circ}\ 56'\ 53'',9;$$

Log. sin. $\tfrac{1}{2}$S = 8,2188003.

H

$$\text{Log.} \frac{2 \text{ sin.}^2 \frac{1}{2}S}{\text{cosin. } \lambda \text{ cosin } \lambda'} = 6,7386383;$$

$$\frac{2 \text{ sin.}^2 \frac{1}{2}S}{\text{cosin. } \lambda \text{ cosin. } \lambda'} = 0,0005478,2.$$

$$\text{Cosin. } \Sigma = 0,9994522,65.$$

	°	′	″
$+M = -4$		3	22,1
$-E' = +2$		59	29,5
$+E = +2$		57	39,7
$-\Sigma = -1$		53	47,1
$\omega' =$	0	0	0,0

From the above example it appears, that a difference, in the errors of two obſervations at above five months diſtance, of 20″, produces an error in the radius vector of little more than $\frac{3}{100}$ of the Earth's mean diſtance from the Sun. Now as the difference of the errors of two accurate obſervations does not exceed 15″ or 16″, from the known accuracy of Mr Slop, we may conclude in the ſuppoſition of our Planet's orbit being circular, that its dif-

ſtance from the Sun is nearly 19,0144342, and that conſequently its periodical revolution is 82 years and about 11 months.

As the angle Σ during an interval of five months is exceeded by S by only $\frac{7}{10}$ of a ſecond, a quantity which has very little influence in the value of the radius vector, it will be unneceſſary in other examples of this Planet to calculate Σ. Which circumſtance, together with taking, by a ſimple proportion, the logarithms of the coſines of the heliocentric latitudes, reduces the calculation to very great ſimplicity.

Let us now take the obſervations of the 6th of October and 13th March. We have as follows:

		d.	h.	′	″
1781.	October	6.	17	7	51;
1782.	March	13.	6	28	58.
	Difference	157.	13	21	7.

$t'' = 13612867''$.

Log. $t'' = 7{,}1339494$;

$\odot = 6^{s}\ 14^{\circ}\ 14'\ 58'',9$;

$\odot' = 11\ 23\ 18\ 47{,}6$.

Log. $a = 9{,}9992284$;

Log. $a' = 9{,}$[illegible]79059.

Sum of the aberration and nutation for Oct. 6. $= -8{,}4$; and for March 13. $= -8{,}0$.

Therefore

$P = 3^{s}\ 2^{\circ}\ 54'\ 34'',7$;

$P' = 2\ 28\ 50\ 43{,}1$;

$M = -0\ 4\ 3\ 51{,}6$;

$T = 3\ 11\ 20\ 24{,}2$.

$T' = 8\ 24\ 28\ 4{,}5$.

Log. sin. $T = 9{,}9914376$.

Log. sin. $T' = 9{,}9979723$.

Log. cosin. $\lambda = 9{,}9999965$.

Log. cosin. $\lambda' = 9{,}9999958$.

Log. sin. $E = 9{,}9906695 -$ log. R.

Log. sin. $E' = 9{,}9958824 -$ log. R.

Log. S $= 5{,}7474428 -$ log. $R\sqrt{R}$.

Let us suppose

$R = 19{,}0144342$,

we have

Log. R $= 1{,}2790834$;

Log. $R\sqrt{R} = 1{,}9186251$.

Log. sin. $E = 8{,}7115861$.

Log. sin. $E' = 8{,}7167990$.

Log. S $= 3{,}8288177$.

therefore

$$\begin{array}{rrrr} & ^{\circ} & ' & '' \\ +M= & -4 & 3 & 51,6 \\ -E'= & +2 & 59 & 10,3 \\ +E= & +2 & 57 & 1,9 \\ -\Sigma= & -1 & 52 & 21,8 \\ \hline \omega= & -0 & 0 & 1,2 \end{array}$$

Let

R=19,008,

we have

Log. R =1,2789364.

Log. R$\sqrt{R}$=1,91840464.

Log. fin. E=8,7117331.

Log. fin. E'=8,7169460.

Log. S =3,8290382.

therefore

$$\begin{array}{rrrr} & ^{\circ} & ' & '' \\ +M= & -4 & 3 & 51,6 \\ -E'= & +2 & 59 & 13,9 \\ +E= & +2 & 57 & 5,5 \\ -\Sigma= & -1 & 52 & 25,2 \\ \hline \omega'= & +0 & 0 & 2,6 \end{array}$$

The proportion gives

$R \quad = 19{,}0124.$

$\text{Log. } R \quad = 1{,}2790369.$

$\text{Log. } R\sqrt{R} = 1{,}9185554.$

$\text{Log. fin. } E = 8{,}7116326.$

$\text{Log. fin. } E' = 8{,}7168455.$

$\text{Log. } S \quad = 3{,}8288874.$

	°	′	″
$+M =$	-4	3	51,6
$-E' =$	$+2$	59	11,4
$+E =$	$+2$	57	3,0
$-\Sigma =$	-1	52	22,8
$\omega =$	0	0	0,0

The nine following combinations are ſuch, that an error of 7″ or 8″ in each obſervation cannot produce, in the value of the radius vector, an error of above $\frac{3}{100}$ of the Earth's mean diſtance from the Sun.

The obſervations by Mr SLOP of Oct. 5. 1781, and March 14. 1782, give

R=19,014434.

Oct. 6. 1781, and March 13. 1782,

R=19,012400.

Oct. 5. 1781, and March 3. 1782,

R=19,011196.

Oct. 8. 1781, and March 11. 1782,

R=19,003736.

Oct. 18. 1781, and March 10. 1782,

R=19,001396.

Oct. 24. 1781, and March 4. 1782,

R=18,982314.

Oct. 24. 1781, and March 3. 1782,

R=18,981850.

Oct. 24. 1781. and March 14. 1782,

R=18,980485.

Oct. 24. 1781, and March 13. 1782,

R=18,978509.

The medium of the above combinations gives

R=18,996258.

In some of the four following combinations, an error in one of the observations of less than $\frac{1}{10}$ of a second makes an alteration of $\frac{4}{100}$ in the radius vector. The results shew how much Mr Slop's observations may be depended on.

The observations of

Oct. 5. and 6. 1781, give

R=18,920860.

Dec. 22. and 23. 1781,

R=19,052614.

Aug. 22. and Sept. 29. 1781,

R=19,017040.

Jan 8. and 15. 1782,

R=18,941870.

Taking a medium we have

$$R=18,983096.$$

This, added to the medium of the former combinations, and the sum divided by 2, gives

R, or the distance of the new planet from the Sun, =18,989677;

that is, nearly *two thousand millions of miles.*

and

$R\sqrt{R}$, or the time of its periodical revolution, =82,7516.

that is, *eighty-two years* and something more than *nine months.*

H's motion in its orbit is therefore as follows:

			°	′	″
Annual,	-	-	4	21	1,3
Diurnal,	-	-	0	0	42,88
Horary,	-	-	0	0	1,79

The obſervations of 24th October 1781, and 13th March 1782, give for

	s	o	′	″
The Longitude of its Node,	2	13	28	55,4
The Inclination of its Orbit to the Ecliptic,	0	0	48	7,4

Thoſe of 24th October 1781, and 3d March 1782.

	s	o		
Long. Nod. -	2	13	29	59,3
Incl. Orb. . -	0	0	48	11,7

The medium of theſe two gives

	s	o	′	″
Long. Nod. .	2	13	29	27,3
Incl. Orb. - -	0	0	48	9,5

The above theory agrees extremely well with the longitudes obſerved by Mr Slop with the Mural Quadrant. The errors of moſt of them do not come the length of 5″, and none exceed 7″ excepting thoſe of October 18th, January 17th, and February 1ſt. Though this does not warrant

us to ſay that the New Planet's orbit is *circular*, it is beyond diſpute that its diſtance from the ſun differed very little during the ſpace of above five months, and that that diſtance was nearly 18,989677.

Two other Series of good obſervations, at the diſtance of two or three years from that which we have made uſe of, and from each other, will give us by a ſimilar procedure two other radii vectores.

By means of the three radii vectores thus found, we ſhall be able in a few years to determine with tolerable accuracy all the elements of the *elliptical* orbit of our Planet.

Though the inclination of its orbit above determined agrees pretty well with the latitudes obſerved during the five months referred to, it cannot be expected to be very exact. From a compariſon of the latitudes of Auguſt and September

1781 with thofe of Auguft and September 1782, the Planet feems to have advanced in latitude fomething more than 3′. Whence it would appear that the inclination of its orbit furpaffes one degree.

The longitudes of Auguft and September 1781, compared with thofe of Auguft and September 1782, fhew that the Planet's velocity is increafing. It would feem, therefore, to be advancing towards the perihelion.

SECTION VII.

Mr SLOP's *Theory*, &c.

FROM two interpolated places, the one 12 h. before, and the other 12 h. after, the Planet's oppofition to the Sun on December 21ft 1781, Mr SLOP found

The Radius Vector=18,9894

and, from a comparison of the observations of 15th May, 17th July, and 18th October 1781 with those of 13th and 14th March 1782,

Long. Nod. $= 2^s\ 14^\circ\ 24'\ 20''$,
Incl. Orb. $= 0\ \ 0\ \ 51\ \ 41$.

This theory gives for the errors of the observations made at Pisa from 22d August 1781 to 14th March 1782 inclusive, as follows:

1781.		*Err. in Long.* ′ ″	*Err. in Lat.* ′ ″
Aug.	22.	+0 34,7	—0 7,7
	23.	+0 28,4	—0 11,7
	24.	+0 29,3	—0 17,8
	26.	+0 31,0	—0 10,0
	27.	+0 31,4	—0 6,6
	28.	+0 26,3	—0 9,4
	29.	+0 28,3	—0 10,1
	30.	+0 30,9	—0 9,3
Sept.	1.	+0 24,2	—0 5,9
	2.	+0 20,2	—0 6,1
	5.	+0 20,7	—0 7,0
	11.	+0 20,4	—0 6,0
	15.	+0 15,6	—0 6,1
	18.	+0 9,0	—0 12,4
	24.	+0 11,7	—0 8,6
	29.	+0 8,3	—0 8,0

1781.	Err. in Long. ′ ″		Err. in Lat. ′ ″	
Nov. 11.	+0	3,4	+0	6,5
Oct. 5.	+0	6,9	—0	11,9
6.	+0	7,1	—0	4,5
8.	+0	5,9	—0	3,7
9.	+0	5,3	—0	8,4
18.	+0	11,8	—0	3,3
24.	+0	1,3	—0	2,9
Dec. 4.	—0	4,6	—0	2,6
6.	—0	4,9	—0	2,8
8.	—0	4,4	—0	7,8
13.	—0	2,7	—0	7,4
14.	—0	0,1	—0	8,6
22.	+0	2,0	—0	2,3
23.	+0	2,0	—0	0,9
27.	—0	1,7	—0	0,9
28.	—0	6,7	—0	2,3
1782.				
Jan. 1.	—0	2,0	—0	3,8
2.	—0	7,5	—0	4,2
4.	—0	1,1	—0	2,8
6.	—0	1,3	—0	3,9
8.	+0	0,7	—0	3,5
9.	+0	5,7	—0	2,3
11.	+0	2,3	—0	3,2
12.	+0	1,1	—0	0,1
13.	+0	3,4	—0	2,1
14.	+0	1,6	—0	1,5
15.	—0	0,5	—0	0,9

1782.		Err. in Long.		Err. in Lat.	
		′	″	′	″
Jan.	17.	—0	11,0	+0	2,0
	19.	—0	3,9	—0	0,6
	20.	+0	3,4	—0	3,1
	22.	+0	0,2	—0	2,6
	28.	—0	1,7	—0	2,7
Feb.	1.	+0	9,5	—0	1,4
	4.	+0	1,9	+0	0,3
	11.	+0	0,1	+0	1,4
	15.	+0	2,1	—0	1,3
	16.	+0	3,8	—0	3,3
	18.	—0	0,5	—0	2,8
	19.	+0	0,7	—0	2,8
	20.	+0	4,4	—0	0,2
	26.	+0	5,6	—0	0,8
	27.	—0	6,0	+0	3,5
Mar.	3.	+0	1,2	+0	1,0
	4.	+0	1,2	+0	2,3
	5.	—0	0,4	+0	1,0
	6.	—0	4,9	—0	2,5
	8.	+0	5,2	+0	4,1
	10.	—0	5,7	+0	3,7
	11.	—0	2,5	+0	1,0
	13.	—0	5,7	—0	1,4
	14.	—0	6,9	0	0,0

THE END.

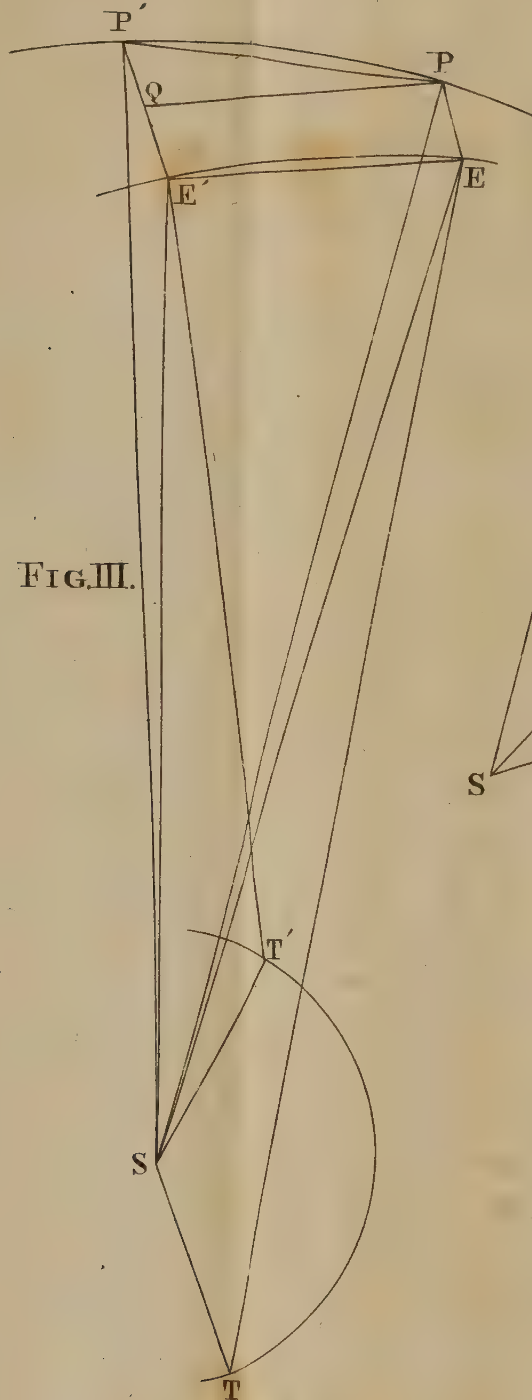

FIG. III.

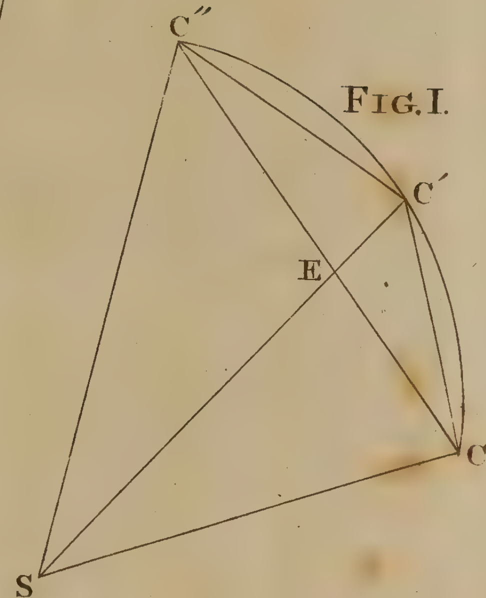

FIG. I.

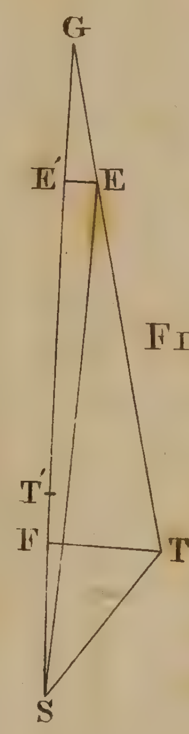

FIG. II.

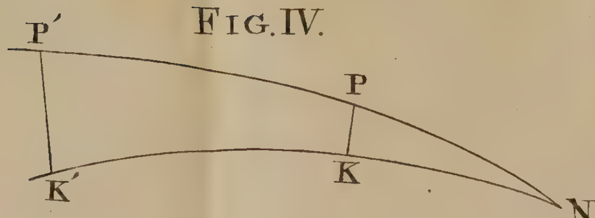

FIG. IV.